Performance of Alamouti scheme with transmit antenna selection

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The bit error rate of the Alamouti scheme with transmit antenna selection in flat Rayleigh fading channels is presented. Performance analysis reveals that this scheme achieves a full diversity order, as if all the transmit antennas were used. This scheme has a fixed low decoding complexity and provides a systematic method to construct full-rate space-time block codes with a full diversity order.

Introduction: Space-time block code (STBC) [1] is a simple scheme that achieves a full diversity order and enjoys the simple maximum likelihood decoding algorithm. As a special case with two transmit antennas, the Alamouti scheme [2] has been proposed for both the W-CDMA and CDMA-2000 standards. Owing to the size and power limitation, most of the current hand-held devices can accommodate only one or at most two antennas. Therefore STBCs designed for a large number of transmit antennas have to be employed to achieve a high diversity order. However, the STBCs could not achieve a full code rate for more than two transmit antennas with complex constellations. Even for real signal constellations, full rate can be maintained only for particular numbers of transmit antennas [1]. Compared with the receive diversity maximal-ratio combining (MRC) scheme of the same diversity order, STBCs incur signal-to-noise ratio (SNR) loss owing to the power spreading across transmit antennas. This SNR loss increases as the number of transmit antennas increases.

To address the issue of difficulty of STBC design and SNR loss, the combination of transmit antenna selection (TAS) with the Alamouti scheme is considered. Two transmit antennas, which maximise the SNR at the receiver, are chosen out of all the available transmit antennas to transmit the Alamouti scheme. We refer to this scheme as TAS/STBC. This scheme is first considered in [3] and improvement is evaluated in terms of average SNR and outage probability. However, the impact of antenna selection on the system performance is not directly investigated. In this Letter, the bit error rate (BER) of the TAS/STBC scheme for binary phase-shift keying (BPSK) in flat Rayleigh fading channels is presented. By the performance analysis, we explicitly prove that the TAS/STBC scheme achieves a full diversity order asymptotically.

System and channel model: We consider an (n_T, n_R) wireless link in a flat Rayleigh fading environment equipped with n_T transmit and n_R receive antennas. The fading coefficients $h_{i,j}$, $1 \le i \le n_T$, $1 \le j \le n_R$, are modelled as independent samples of complex Gaussian random variables with a zero mean and the variance of 0.5 per dimension. It is assumed that the channel state information is perfectly known at the receiver and partially known at the transmitter through a feedback channel. The $(n_T, 2; n_R)$ TAS/STBC scheme with the Alamouti scheme as the baseline code is considered. At any time, only two out of n_T antennas are chosen and activated for transmission of the Alamouti scheme. All the n_R receive antennas are used without selection. The two transmit antennas, labelled as U and V, are chosen based on the calculation

$$(U, V) = \underset{1\le u, v\le n_T, u\ne v}{\operatorname{argmax}} \left\{ \sum_{j=1}^{n_R} (|h_{u,j}|^2 + |h_{v,j}|^2) \right\} \tag{1}$$

BER expression: The BER expression for the BPSK TAS/STBC in flat Rayleigh fading channels is derived and presented in (2). The details of derivation are reported in other publications.

$$\begin{aligned}
P_2 = \frac{2n_T(n_T-1)}{[(n_R-1)!]^2} \Bigg\{ & \sum_{i=0}^{n_T-2} \sum_{j=0}^{n_R-1} (-1)^{i+1} \binom{n_T-2}{i} j! \binom{n_R-1}{j} \\
& \times \sum_{t=0}^{(n_R-1)i} a_t(n_R, i) \sum_{k=0}^{2n_R+t-j-2} \frac{k! \binom{2n_R+t-j-2}{k}}{2^{2n_R+t-j-k-2}(i+2)^{2n_R+t-j-1}} \\
& \times \left[(2n_R+t-j-k-2) W(2n_R+t-j-k-2, i+2) - \frac{1}{2} W(2n_R+t-j-k-1, i+2) \right] \\
& - \sum_{j=0}^{n_R-1} j! \binom{n_R-1}{j} \sum_{p=0}^{n_R-j-1} \frac{(-1)^p \binom{n_R-j-1}{p}}{2^{3n_R+p-j-1}(n_R+p)} \\
& \times \left[(2n_R-j-1) W(2n_R-j-1, 2) - \frac{1}{2} W(2n_R-j, 2) \right] \\
& + \sum_{i=1}^{n_T-2} \sum_{j=0}^{n_R-1} (-1)^i \binom{n_T-2}{i} j! \binom{n_R-1}{j} \sum_{t=0}^{(n_R-1)i} a_t(n_R, i) \\
& \times \sum_{p=0}^{n_R-j-1} (-1)^p \binom{n_R-j-1}{p} \\
& \times \sum_{k=0}^{n_R+p+t-1} \frac{k! \binom{n_R+p+t-1}{k}}{2^{n_R+p+t-k-1}(i+2)^{2n_R+t-j-k-2} i^{k+1}} \\
& \times \left[(2n_R+t-j-k-2) W(2n_R+t-j-k-2, i+2) - \frac{1}{2} W(2n_R+t-j-k-1, i+2) \right] \\
& - \sum_{i=1}^{n_T-2} \sum_{j=0}^{n_R-1} (-1)^i \binom{n_T-2}{i} j! \binom{n_R-1}{j} \sum_{t=0}^{(n_R-1)i} a_t(n_R, i) \\
& \times \sum_{p=0}^{n_R-j-1} \frac{(-1)^p \binom{n_R-j-1}{p} (n_R+p+t-1)!}{2^{n_R-j-p-1} i^{n_R+p+t}} \\
& \times \left[(n_R-j-p-1) W(n_R-j-p-1, 2) - \frac{1}{2} W(n_R-j-p, 2) \right] \Bigg\}
\end{aligned} \tag{2}$$

In (2), $a_t(n_R, i)$ is the coefficients of z^t, $0 \le t \le i(n_R-1)$, in the expansion

$$\left(\sum_{k=0}^{n_R-1} \frac{z^k}{k!} \right)^i \tag{3}$$

and $W(n, k)$ is defined as

$$W(n, k) = \frac{(n-1)!}{2} \left(1 - \sqrt{\frac{\gamma}{\gamma+k}} \right)^n \times \sum_{q=0}^{n-1} \binom{n+q-1}{q} \left[\frac{1}{2} \left(1 + \sqrt{\frac{\gamma}{\gamma+k}} \right) \right]^q \tag{4}$$

in which $W(n, k) = 0$ if $n = 0$, and $\gamma = E_b/N_0$, where E_b is the energy per bit at the transmitter and N_0 is the spectral density of the additive white Gaussian noise (AWGN) at each receive antenna.

Next we investigate the $(n_T, 2; 1)$ TAS/STBC, in which there is only one receive antenna. For $n_R = 1$, we have $a_t(n_R = 1, i) = 1$ from (3) and (2) can be simplified as

$$\begin{aligned}
P_2 = \frac{n_T(n_T-1)}{2} \Bigg\{ & \sum_{i=0}^{n_T-2} \frac{(-1)^i \binom{n_T-2}{i}}{i+2} \left(1 - \sqrt{\frac{\gamma}{\gamma+i+2}} \right) \\
& - \sum_{i=1}^{n_T-2} \frac{(-1)^i \binom{n_T-2}{i}}{i} \left(1 - \sqrt{\frac{\gamma}{\gamma+i+2}} \right) \\
& + \left[\sum_{i=1}^{n_T-2} \frac{(-1)^i \binom{n_T-2}{i}}{i} - \frac{1}{2} \right] \left(1 - \sqrt{\frac{\gamma}{\gamma+2}} \right) \\
& + \frac{1}{4} \left(1 - \sqrt{\frac{\gamma}{\gamma+2}} \right)^2 \left(2 + \sqrt{\frac{\gamma}{\gamma+2}} \right) \Bigg\}
\end{aligned} \tag{5}$$

Considering the asymptotic case when $\gamma \to \infty$, we have

$$\lim_{\gamma\to\infty} P_2 \gamma^{n_T} = \frac{(2n_T-1)!}{2^{2n_T-1}(n_T-1)!} \tag{6}$$

This expression means that if $\gamma \gg 1$, (5) can be approximated as

$$P_2 \simeq \frac{(2n_T - 1)!}{2^{2n_T - 1}(n_T - 1)!}\left(\frac{1}{\gamma}\right)^{n_T} \tag{7}$$

which shows that a full diversity order of n_T is achieved asymptotically for the (n_T, 2; 1) TAS/STBC.

For the (n_T, 2; 2) TAS/STBC, in which there are two receive antennas, we have

$$a_t(n_R = 2, i) = \binom{i}{t}$$

from (3). If $\gamma \gg 1$, (2) can be approximated as

$$P_2 \simeq \frac{n_T(4n_T - 1)!}{2^{5n_T - 2}(2n_T - 1)(2n_T - 1)!}\left(\frac{1}{\gamma}\right)^{2n_T} \tag{8}$$

which indicates that a full diversity order of $2n_T$ is achieved asymptotically for the (n_T, 2; 2) TAS/STBC.

The cases for $n_R \geq 3$ are not of practical significance since it is difficult to put more than two antennas at the mobile set in downlink communications. Therefore, the cases for $n_R \geq 3$ are not presented.

Performance: We illustrate the BER results of the TAS/STBC by simulation for BPSK modulation. Flat Rayleigh fading is assumed.

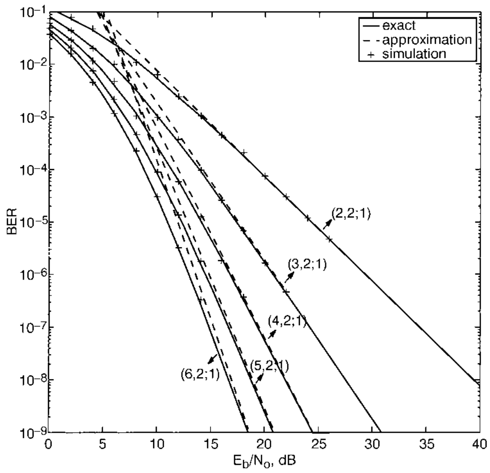

Fig. 1 *Comparison between exact expression, approximation and simulation for (n_T, 2; 1) TAS/STBCs, BPSK*

Fig. 1 shows the comparison between the exact expression in (5), the approximation in (7), and the simulation for (n_T, 2; 1) TAS/STBCs, $2 \leq n_T \leq 6$. Note that the (2, 2; 1) TAS/STBC is the (2, 1) Alamouti scheme without antenna selection. It clearly shows that the theoretical results match the simulation ones. It also shows that (5) asymptotically approaches (7), which is a tight bound of (5) at high SNRs. This means that a full diversity order of 3 up to 6 has been achieved for the (3, 2; 1) up to (6, 2; 1) TAS/STBCs, respectively.

In Fig. 2, we compare the performance of the (2, 2) Alamouti scheme without antenna selection, the (4, 2; 1) TAS/STBC and the (4, 1) STBC [1], all obtained by simulation. Fig. 2 shows that the (4, 1) STBC incurs a 3 dB performance loss compared with the (2, 2) Alamouti scheme. However, this loss can be dramatically decreased by the TAS/STBC. It is evident that the (4, 2; 1) TAS/STBC is 1.9 dB superior to the (4, 1) STBC and only loses around 1.1 dB compared with the Alamouti scheme with two receive antennas, while maintaining the same diversity order. Fig. 2 clearly demonstrates the advantage of TAS/STBCs relative to conventional STBCs in terms of SNR.

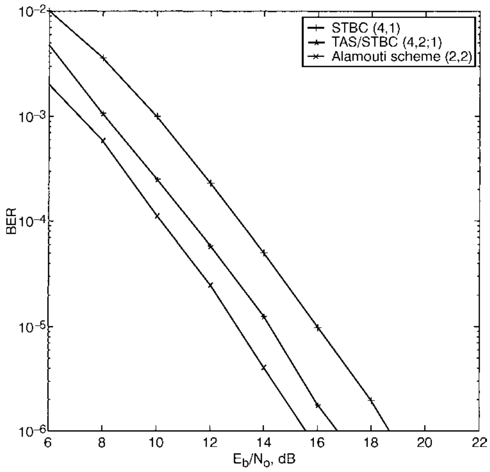

Fig. 2 *Performance comparison between different STBCs with diversity order of 4, BPSK*

From the analysis and simulation, it is shown that the (n_T, 2; n_R) TAS/STBC can achieve a full diversity order of $n_T n_R$. This conclusion is similar to that for other transmit antenna selection schemes in [4] and [5]. The decoding complexity of the (n_T, 2; n_R) TAS/STBCs is the same as that of the (2, n_R) Alamouti scheme. Therefore, the TAS/STBC scheme is suitable for downlink communications links and provides a systematic method to construct STBCs of a high diversity order with simple decoding complexity. Full rate can always be maintained even for complex constellations and any diversity order. For a specified diversity order as the system design target, no particular code design is involved and only the number of transmit antennas to be chosen from needs to be adjusted at the base station.

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